

# Sigma and Pion Decays of Mixed Heavy Hybrids

Leonard S. Kisslinger and Dara J. Krute

Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213

## Abstract

We estimate the relative branching ratio of  $\sigma$  to  $2\pi$  for the decay of the  $\Psi'(2S)$ , which we have found to be a mixed charm and hybrid charmonium meson. The external field method and correlators from a previous calculation are used to estimate these decays. We find the  $\sigma$  to  $2\pi$  branching ratio for  $\Psi'(2S)$  to the  $J/\Psi(1S)$  decay to be 0.98, in agreement with an experiment at IHEP-BES.

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## 1 Introduction

In previous work[1] it was shown that the  $\Psi'(2S)$  charmonium and  $\Upsilon(3S)$  bottomonium states are admixed hybrid and normal meson states, with approximately a 50% mixture. For the  $\Upsilon(nS)$  states this provides an explanation for the unusual two-pion decays[3]; and for the  $\Psi'(2S)$  this solves the famous  $\rho-\pi$  problem. Prior to this, we had studied the  $\Psi'(2S)$ , also using the method of QCD sum rules, and found[2] that it was not a pure hybrid. In the present work we estimate the decay of the  $\Psi'(2S)$  to the  $J/\Psi(1S)$  with a sigma or two free pions as a test of our mixed heavy quark hybrid meson theory.

The operator which produces a mixed vector ( $J^{PC} = 1^{--}$ ) charmonium state is

$$J^\mu = bJ_H^\mu + \sqrt{1-b^2}J_{HH}^\mu, \quad (1)$$

with  $b$  a constant and

$$\begin{aligned} J_H^\mu &= \bar{q}_c^a \gamma^\mu q_c^a \\ J_{HH}^\mu &= \bar{q}_c^a \gamma_\nu G^{\mu\nu} q_c^a, \end{aligned} \quad (2)$$

where  $J_H^\mu$  is the standard current for a  $1^{--}$  charmonium state and  $J_{HH}^\mu$  is the heavy charmonium hybrid current, with  $q_c^a$  and  $\gamma_\nu$  the charm quark field and the Dirac matrix, respectively.  $G^{\mu\nu}$  is

$$G^{\mu\nu} = \sum_{a=1}^8 \frac{\lambda_a}{2} G_a^{\mu\nu}, \quad (3)$$

with  $\lambda_a$  the SU(3) generator ( $Tr[\lambda_a \lambda_b] = 2\delta_{ab}$ ) and  $G_a^{\mu\nu}$  is the gluon field.

The method of QCD sum rules[4] uses a correlator, which we now define. Given the operator  $J_A$ , often called a current, which creates the state  $|A\rangle$  from the vacuum,  $|\rangle$ , i.e.,  $J_A|\rangle = |A\rangle$ , the correlator  $\Pi_A(x)$  in coordinate space is defined by

$$\Pi_A(x) = \langle T[J_A(x)J_A(0)] \rangle ,$$

where  $T$  is the time-ordering operator. For the investigation of a  $1^{--}$  charmonium state, the correlators in coordinate space for a normal meson,  $\Pi_H^{\mu\nu}(x)$ , or a hybrid meson,  $\Pi_{HH}^{\mu\nu}(x)$ , are

$$\begin{aligned} \Pi_H^{\mu\nu}(x) &= \langle T[J_H^\mu(x)J_H^\nu(0)] \rangle \\ \Pi_{HH}^{\mu\nu}(x) &= \langle T[J_{HH}^\mu(x)J_{HH}^\nu(0)] \rangle , \end{aligned} \quad (4)$$

For the mixed charmonium state one uses the current  $J^\mu$ , Eq(1). The QCD sum rule method equates a dispersion relation for the correlator to an operator product expansion, with a Borel transform to provide convergence. In the present research we use correlators to obtain matrix elements for decays of a mixed hybrid charmonium state that was found using QCD sum rules, but we do not need the sum rules themselves.

In Ref[1] a QCD sum rule calculation was carried out using  $J^\mu$  as the current for the correlator. It was shown that for both the  $\Psi'(2S)$  and  $\Upsilon(3S)$  states  $b \simeq 0.7$ , so that both of these states are approximately a 50% admixture of normal and hybrid mesons. In the present work we use these currents to estimate the branching ratios for the decay of the  $\Psi'(2S)$  to the  $J/\Psi(1S)$  with either a sigma or two pions, with the sigma being a broad  $2\pi$  resonance with a mass of about 600 Mev.

## 2 $\sigma$ and $2\pi$ Decays of the $\Psi'(2S)$

The mechanism for the  $2\pi$  decay of a normal component of the  $\Psi'(2S)$  charmonium state to the  $J/\Psi(1S)$  normal charmonium state is shown in Fig. 1, with the correlator defined with two currents, as in Eq(4), but with operators for pion production from quarks between the currents. We discuss this below.

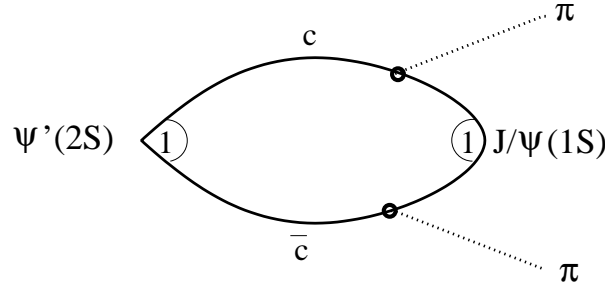


Figure 1: Two free pion decay of a normal charmonium meson

The mechanism for the  $2\pi$  decay of a hybrid charmonium to a charmonium meson is shown in Fig. 2, with the correlator defined using  $J_{HH}^\mu$  and  $J_H^\mu$  currents joined by pion production operators. The mechanism for  $\sigma$  decay of a hybrid charmonium to a charmonium meson is shown in Fig. 3. The correlator differs from that of Fig. 2 in that there are no pion production operators, and the sigma is produced by coupling to the gluon, G.

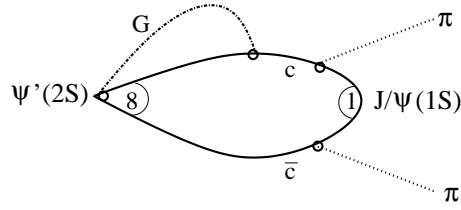


Figure 2: Two-pion decay of a hybrid charmonium meson

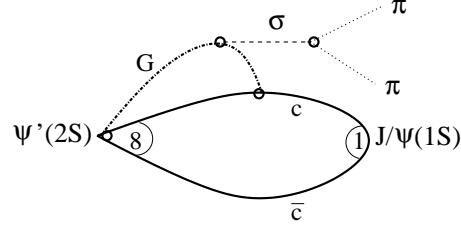


Figure 3: Sigma decay of a hybrid charmonium meson

For the calculation of  $\sigma$  and  $2\pi$  decays of a mixed hybrid state, like the  $\Psi'(2S)$ , we use the external field method[5, 6]. The starting points are the diagrams used for the study of a mixed normal-hybrid state[1] and the underlying diagrams for the decays shown in Figs. 1, 2, and 3. These are shown in Fig. 4.

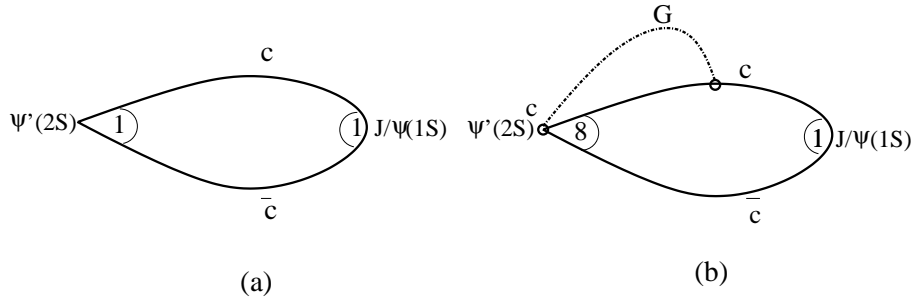


Figure 4: Basic processes for mixed charmonium to couple to normal charmonium

Fig. 4(a) shows the mechanism for normal charmonium coupling, while Fig. 4(b) provides the basic mechanism for hybrid coupling to normal charmonium. Using the usual notation

of QCD sum rules these diagrams are the correlators  $\Pi_H$  (Fig. 4(a)) and  $\Pi_{H-HH}$  (Fig. 4(b)), respectively.

The correlator corresponding to Fig. 4a is

$$\Pi_H^{\mu\nu}(p) = 3g^2 \int \frac{d^4k}{(4\pi)^4} \text{Tr}[S(k)\gamma^\mu\gamma_5 S(p-k)(\gamma^\nu\gamma_5)^T], \quad (5)$$

with

$$S(k) \equiv i \frac{\not{k} + M_c}{k^2 - M_c^2}. \quad (6)$$

Carrying out the traces and the  $d^4k$  momentum integral, one finds

$$\begin{aligned} \Pi_H^{\mu\nu}(p) &= 12g^2[g^{\mu\nu}(M_c^2 I_H(p) - p_\alpha I_H^\alpha(p) + 2p^\mu I_H^\nu(p) - 2I_H^{\mu\nu}(p)], \text{ with} \\ I_H(p) &= \frac{2M_c^2 - p^2/2}{4\pi^2} I_o(p) \\ I_H^\nu(p) &= p^\nu \left[ \frac{2M_c^2 - p^2/2}{8\pi^2} I_o(p) + 7/4 \right] \\ I_H^{\mu\nu}(p) &= \frac{g^{\mu\nu}}{4 \cdot 4\pi^2} ([2M_c^4 - M_c^2 p^2/2 + (p^2/6)(5M_c^2 - p^2 - 4M_c^4/p^2)] I_o(p) + \text{constant}) \\ &\quad + \frac{p^\mu p^\nu}{6 \cdot 4\pi^2} [(5M_c^2 - p^2 - 4M_c^4/p^2) I_o(p) - 4M_c^2/p^2] \\ I_o(p) &= \int_0^1 \frac{d\alpha}{p^2(\alpha - \alpha^2) - M_c^2}. \end{aligned} \quad (7)$$

As in our calculations using QCD sum rules[1, 2] we use the scalar component,  $\Pi_H^S$ , defined as  $\Pi_H^{\mu\nu}(p) = ((p_\mu p_\nu/p^2) - g^{\mu\nu})\Pi^V(p) + (p_\mu p_\nu/p^2)\Pi^S(p)$ , with  $\Pi_H^V(p)$  the vector component. From Eq(6) we obtain

$$\Pi_H^S(p) = \frac{12}{(4\pi)^2} (8M_c^4/3 - 4M_c^2 p^2/3 + p^4/6) I_o(p) + \text{constant} + \text{constant} \cdot p^2. \quad (8)$$

Using the Borel transforms from momentum to the Borel mass,  $M_B$

$$\begin{aligned} \mathcal{B}I_0(p) &= 2e^{-z} K_0(z) \\ \mathcal{B}p^2 I_0(p) &= 4M_c^2 e^{-z} [K_0(z) + K_1(z)] \\ \mathcal{B}p^4 I_0(p) &= 2M_c^2 e^{-z} [6K_0(z) + 8K_1(z) + 2K_2(z)] \end{aligned} \quad (9)$$

with  $z = 2M_c^2/M_B^2$  and  $K_n$  are modified Bessel functions. Therefore we find

$$\Pi_H^S(M_B) = \frac{4}{(4\pi)^2} g^2 e^{-z} [6K_0(z) - 8K_1(z) + 2K_2(z)] \quad (10)$$



Figure 5: (a) Pion coupled to quark (b) Sigma coupled to gluon decays to two pions

To find the two-pion decay of the  $\Psi'(2S)$  we use the external field method[5, 6] with the quark propagating with an external pion,  $S_\pi(p)$ , as shown in Fig 5(a),

$$S_\pi(p) = ig_\pi \tau \cdot \phi_\pi \frac{1}{\not{p} - M} \gamma_5 \frac{1}{\not{p} - M}, \quad (11)$$

where  $g_\pi$  is the pion-quark coupling constant and  $\phi_\pi$  is the pion field.

From this we obtain the two-pion decay of the normal component of  $\Psi'(2S)$ , corresponding to Fig. 1 as

$$\begin{aligned} \Pi_{H\pi\pi}^{\mu\nu} &= -3g_\pi^2 g^2 \int \frac{d^4k}{(4\pi)^4} \frac{\text{Tr}[\gamma^\mu \gamma^5 \gamma^\nu \gamma^5]}{(k^2 - M_c^2)((p-k)^2 - M_c^2)} \\ &= \frac{12}{(4\pi)^2} g^{\mu\nu} g_\pi^2 g^2 [(2M_c^2 - p^2/2)I_0(p) + \text{constant}], \end{aligned} \quad (12)$$

from which one obtains, after a Borel transform, for the scalar component of the two-pion correlator

$$\Pi_{H\pi\pi}^S(M_B) = \frac{24}{(4\pi)^2} g_\pi^2 g^2 M_c^2 e^{-z} [K_0(z) - K_1(z)]. \quad (13)$$

The correlator  $\Pi_{H-HH\pi\pi}^S(M_B)$  for  $2\pi$  decay from the hybrid component of the  $\Psi'(2S)$  to the  $J/\Psi(1S)$  state, obtained from the diagram in Fig.2, turns out to be less than 1 % of  $\Pi_{H\pi\pi}^S(M_B)$  of Fig. 1 and is not included.

Corresponding to the diagram shown in Fig.3, the correlator for the hybrid component of  $\Psi'(2S)$  to decay to a sigma and the  $J/\Psi(1S)$  state,  $\Pi_{H-HH\sigma}^{\mu\nu}(p)$ , is obtained using two results from previous work. First, it was shown in Ref[1] that the coupling of the mixed to the normal charmonium,  $\Pi_{H-HH}^S(M_B)$ , shown in Fig. 4(b), is given approximately by

$$\Pi_{H-HH}^S(M_B) \simeq \pi^2 \Pi_H^S(M_B), \quad (14)$$

where  $\Pi_H^S(M_B)$  is the coupling of the normal component of  $\Psi'(2S)$  to the  $J/\Psi(1S)$  state.

Next we use the external field coupling shown in Fig.5(b) to relate  $\Pi_{H-HH\sigma}^{\mu\nu}(p)$  to the mixed coupling,  $\Pi_{H-HH}^S(M_B)$  which gives

$$\begin{aligned} \Pi_{H-HH\sigma}^S &= \frac{g_\sigma}{M_\sigma} \langle G^2 \rangle \Pi_{H-HH}^S \\ &= \frac{g_\sigma}{M_\sigma} \langle G^2 \rangle \pi^2 \Pi_H^S, \end{aligned} \quad (15)$$

from which we find

$$\Pi_{H-HH\sigma}^S(M_B) = \frac{24}{(4\pi)^2} g^2 \frac{g_\sigma}{M_\sigma} < G^2 > M_c^2 \pi^2 e^{-z} [K_0(z) - 4K_1(z)/3 + K_2(z)/3], \quad (16)$$

where  $g_\sigma$  is the sigma-gluon coupling constant and  $< G^2 >$  is the gluon condensate.

From Eqs(13,16) we obtain  $R$  = ratio of sigma to two-pion decay as

$$R = N \frac{\frac{g_\sigma}{M_\sigma} < G^2 > M_c^2 \pi^2 [K_0(z) - 4K_1(z)/3 + K_2(z)/3]}{g_\pi^2 [K_0(z) - K_1(z)]}, \quad (17)$$

with  $N$  a normalization factor derived below. The gluon-sigma coupling constant,  $g_\sigma$ , was found using a low-energy theorem[7] and has been used successfully in studying scalar glueballs[8], the production of sigmas in high-energy proton-proton collisions[9], and the decay of hybrid baryons[10]. The result is  $g_\sigma/M_\sigma \simeq 1.0$ . The pion quark coupling is known to be[11]  $g_\pi = f_\pi m_\pi^2/(m_u + m_d)$ . Using standard values for the quark masses,  $m_u = m_d = 3$  MeV,  $g_\pi^2 = 0.189 \text{ GeV}^4$ . The gluon condensate  $< G^2 > = 0.476 \text{ GeV}^4$ .

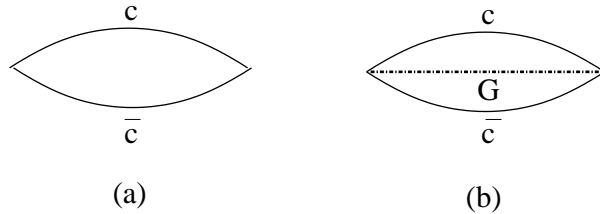


Figure 6: (a) Normal charmonium normalization (b) Hybrid charmonium normalization

Finally, since the correlators do not use normalized states we must find the relative normalization of the normal and hybrid components. The normalization parameter is found from

$$N^2 = \frac{\int d(M_B^2) \Pi_H^S(M_B)}{\int d(M_B^2) \Pi_{H-HH}^S(M_B)}. \quad (18)$$

Using Eq(10) for  $\Pi_H^S(M_B)$  and Eq(15) in Ref[2] for  $\Pi_{H-HH}^S(M_B)$  we find that  $N=0.0123 M_c^2$ . Using the charmonium mass,  $M_c$ , and the Borel mass,  $M_B$ =mass of the  $\Psi'(2S)$ , and  $(K_0(z), K_1(z), K_2(z))=(1.54, 3.75, 31.53)$ , we find for  $R$  = ratio of  $\sigma$  to  $2\pi$  decay of the  $\Psi'(2S)$  to the  $J/\Psi$

$$R = 0.98. \quad (19)$$

This result is in very good agreement with recent measurements by the BES Collaboration at the IHEP, Beijing, [12]. The uncertainty in the  $R$  is not large, similar to the rapid convergence of diagrams used in QCD sum rules for heavy quark states. Since we are using only the lowest order diagram, from these previous calculations we estimate an uncertainty of about 15 percent.

### 3 Conclusion

Using the results of our previous study that found the  $\Psi'(2S)$  to be a mixed normal and hybrid charmonium state, we have found the branching ratio of the sigma to two-pion decays of this state to the  $J/\Psi$  to be in good agreement with BES measurements. With the successful application of this theory to explain the so-called  $\rho - \pi$  decay problem of the  $\Psi'(2S)$ [1], this gives further evidence that the  $\Psi'(2S)$  is a mixed heavy-quark hybrid. In future studies we shall explore other decays, and the production of the  $\Psi'(2S)$  in Relativistic Heavy Ion Collisions, where the octet model has been shown to be dominant[13, 14], consistent with the color 8 gluon of the hybrid component.

### 4 Acknowledgements

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### References

- [1] L.S. Kisslinger, Phys. Rev. **D 79**, 114026 (2009)
- [2] L. S. Kisslinger, D. Parno, S. Riordan, Adv.High Energy Phys. 2009:982341,2009
- [3] H. Vogel, hep-ex/060601, Proceedings of 4th Flavor Physics and CP Violation Conference (FPCP'06) (2006), [www.slac.stanford.edu/econf/C060409](http://www.slac.stanford.edu/econf/C060409)
- [4] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147**, 385; Nucl. Phys. **B147** 448 (1979)
- [5] C.B. Chiu, J. Pasupathy, and S.J. Wilson, Phys. Rev. **D32**, 1786 (1985)
- [6] E.M. Henley, W-Y.P. Hwang, and L.S. Kisslinger, Phys. Lett. **B367**, 21 (1996)
- [7] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B165** 67 (1980); Nucl. Phys. **B191** 301 (1981)
- [8] L.S Kisslinger and M.B. Johnson, Phys. Lett. **B523**, 127 (2001)
- [9] L.S. Kisslinger, W-h. Ma and P. Shen, Phys. Rev. **D 71**, 094021 (2005)
- [10] L.S. Kisslinger and Z. Li, Phys. Lett. **B445**, 271 (1999)
- [11] V.M. Dubovik and S.V.Zenkin, Ann. Phys. (NY) **172**, 100 (1986)
- [12] BES Collaboration, Phys. Lett. **B645** 19 (2007)

- [13] G.C. Nayak, M.X. Liu, and F. Cooper, Phys. Rev. **D 68**, 034003 (2003)
- [14] F. Cooper, M.X. Liu, and G.C. Nyak, Phys. Rev. Lett. **93**, 171801 (2004)



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## 1 Introduction

For many years there have been experimental and theoretical studies of the decay of the  $\Psi'(2S)$  charmonium state. First, it was found that the hadronic decays of the  $\Psi'(2S)$  compared to the  $J/\Psi(1S)$  were an order of magnitude smaller than predicted by QCD perturbation theory[1]. This was quite surprising, as for heavy quark systems the lowest order terms should dominate. It is called the  $\rho - \pi$  problem. More recently, studies of final state interaction in the  $2\pi$  decay of the  $\Psi'(2S)$  vs. the  $J/\Psi(1S)$  were carried out[2]. This is directly related to our present work on  $\sigma$  vs  $2\pi$  decay, since the  $\sigma$  is a  $2\pi$  resonance (see, e.g., Ref [3]). Our approach, however, is quite different, since it is based on the well-known strong coupling of the  $\sigma$  to a gluon, as discussed in detail below. See Ref [2] for references to earlier theoretical studies related to  $2\pi$  decay of heavy quarkonium systems.

In previous work[4] it was shown that the  $\Psi'(2S)$  charmonium and  $\Upsilon(3S)$  bottomonium states are admixed hybrid and normal meson states, with approximately a 50% mixture. For the  $\Upsilon(nS)$  states this provides an explanation for the unusual two-pion decays[6]; and for the  $\Psi'(2S)$  this solves the famous  $\rho - \pi$  problem. Prior to this, we had studied the  $\Psi'(2S)$ , also using the method of QCD sum rules, and found[5] that it was not a pure hybrid. In the present work we estimate the decay of the  $\Psi'(2S)$  to the  $J/\Psi(1S)$  with a sigma or two free pions as a test of our mixed heavy quark hybrid meson theory.

The operator which produces a mixed vector ( $J^{PC} = 1^{--}$ ) charmonium state is

$$J^\mu = bJ_H^\mu + \sqrt{1-b^2}J_{HH}^\mu, \quad (1)$$

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there are no contributions with both pions coupled to the  $c$  or  $\bar{c}$ . In the figures the 1 at a vertex means a color singlet, while an 8 is a color octet. Since a gluon is a color octet, a hybrid meson consists of quarks in an octet coupled with a gluon to form a color singlet, as in the second line of Eq(2).  $G - \pi$  coupling vanishes.

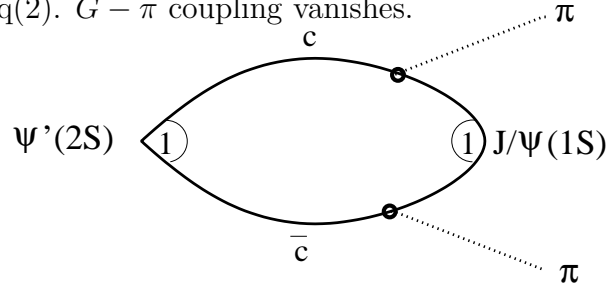


Figure 1: Two free pion decay of a normal charmonium meson

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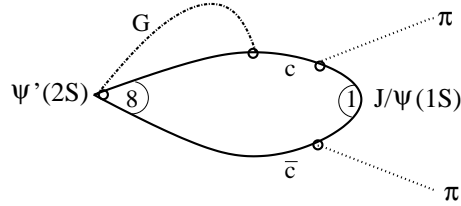


Figure 2: Two-pion decay of a hybrid charmonium meson

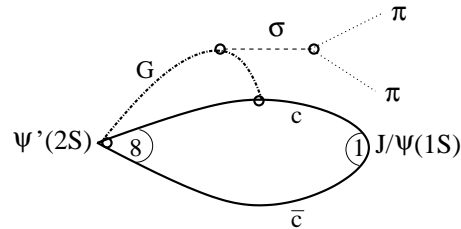


Figure 3: Sigma decay of a hybrid charmonium meson

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1, 2, and 3. The coupling of mixed hybrid charmonium to normal charmonium is shown in Figs. 4(a) and 4(b).

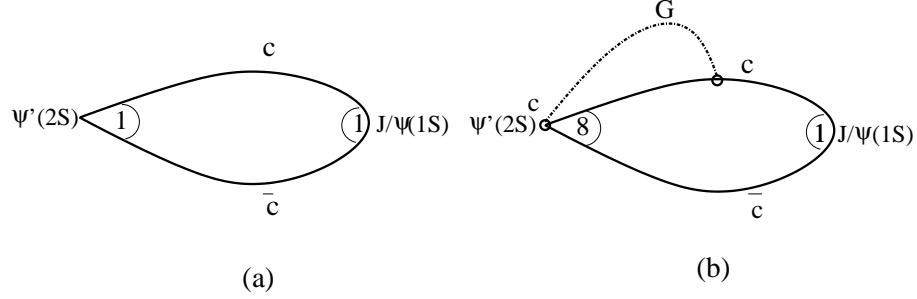


Figure 4: Basic processes for mixed charmonium to couple to normal charmonium

Fig. 4(a) shows the mechanism for normal charmonium coupling, while Fig. 4(b) provides the basic mechanism for hybrid coupling to normal charmonium. Using the usual notation of QCD sum rules these diagrams are the correlators  $\Pi_H$  (Fig. 4(a)) and  $\Pi_{H-HH}$  (Fig. 4(b)), respectively.

The correlator corresponding to Fig. 4(a) is

$$\Pi_H^{\mu\nu}(p) = 3g^2 \int \frac{d^4k}{(4\pi)^4} \text{Tr}[S(k)\gamma^\mu\gamma_5 S(p-k)(\gamma^\nu\gamma_5)^T], \quad (5)$$

with

$$S(k) \equiv i \frac{\not{k} + M_c}{k^2 - M_c^2}. \quad (6)$$

Carrying out the traces and the  $d^4k$  momentum integral, one finds

$$\begin{aligned} \Pi_H^{\mu\nu}(p) &= 12g^2[g^{\mu\nu}(M_c^2 I_H(p) - p_\alpha I_H^\alpha(p)) + 2p^\mu I_H^\nu(p) - 2I_H^{\mu\nu}(p)], \text{ with} \\ I_H(p) &= \frac{2M_c^2 - p^2/2}{4\pi^2} I_o(p) \\ I_H^\nu(p) &= p^\nu \left[ \frac{2M_c^2 - p^2/2}{8\pi^2} I_o(p) + 7/4 \right] \\ I_H^{\mu\nu}(p) &= \frac{g^{\mu\nu}}{4 \cdot 4\pi^2} ([2M_c^4 - M_c^2 p^2/2 + (p^2/6)(5M_c^2 - p^2 - 4M_c^4/p^2)] I_o(p) + \text{constant}) \\ &\quad + \frac{p^\mu p^\nu}{6 \cdot 4\pi^2} [(5M_c^2 - p^2 - 4M_c^4/p^2) I_o(p) - 4M_c^2/p^2] \\ I_o(p) &= \int_0^1 \frac{d\alpha}{p^2(\alpha - \alpha^2) - M_c^2}. \end{aligned} \quad (7)$$

As in our calculations using QCD sum rules[4, 5] we use the scalar component,  $\Pi_H^S$ , defined as  $\Pi_H^{\mu\nu}(p) = ((p^\mu p^\nu/p^2) - g^{\mu\nu})\Pi_H^V(p) + (p^\mu p^\nu/p^2)\Pi_H^S(p)$ , with  $\Pi_H^V(p)$  the vector component. The

vector component provides a separate sum rule, but the sum rule with the scalar component has less error (as determined from the method of QCD sum rules in previous calculations-see, e.g., Refs[5, 4]). From Eq(6) we obtain

$$\Pi_H^S(p) = \frac{12}{(4\pi)^2}(8M_c^4/3 - 4M_c^2p^2/3 + p^4/6)I_0(p) + \text{constant} + \text{constant} \cdot p^2. \quad (8)$$

Using the Borel transforms from momentum to the Borel mass,  $M_B$

$$\begin{aligned} \mathcal{B}I_0(p) &= 2e^{-z}K_0(z) \\ \mathcal{B}p^2I_0(p) &= 4M_c^2e^{-z}[K_0(z) + K_1(z)] \\ \mathcal{B}p^4I_0(p) &= 2M_c^4e^{-z}[6K_0(z) + 8K_1(z) + 2K_2(z)] \end{aligned} \quad (9)$$

with  $z = 2M_c^2/M_B^2$  and  $K_n$  are modified Bessel functions. Therefore we find

$$\Pi_H^S(M_B) = \frac{4M_c^4}{(4\pi)^2}g^2e^{-z}[6K_0(z) - 8K_1(z) + 2K_2(z)] \quad (10)$$

To find the two-pion decay of the  $\Psi'(2S)$  we use the external field method[8, 9] with the quark propagating with an external pion,  $S_\pi(p)$ , as shown in Fig 5(a),

$$S_\pi(p) = ig_\pi\tau \cdot \phi_\pi \frac{1}{\not{p} - M} \gamma_5 \frac{1}{\not{p} - M}, \quad (11)$$

where  $g_\pi$  is the pion-quark coupling constant and  $\phi_\pi$  is the pion field.

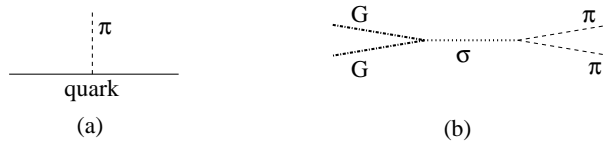


Figure 5: (a) Pion coupled to quark (b) Sigma coupled to gluon decays to two pions

From this we obtain the two-pion decay of the normal component of  $\Psi'(2S)$ , corresponding to Fig. 1 as

$$\begin{aligned} \Pi_{H\pi\pi}^{\mu\nu} &= -3g_\pi^2g^2 \int \frac{d^4k}{(4\pi)^4} \frac{\text{Tr}[\gamma^\mu\gamma^5\gamma^\nu\gamma^5]}{(k^2 - M_c^2)((p-k)^2 - M_c^2)} \\ &= \frac{12}{(4\pi)^2}g^{\mu\nu}g_\pi^2g^2[(2M_c^2 - p^2/2)I_0(p) + \text{constant}], \end{aligned} \quad (12)$$

from which one obtains, after a Borel transform, for the scalar component of the two-pion correlator

$$\Pi_{H\pi\pi}^S(M_B) = \frac{24}{(4\pi)^2}g_\pi^2g^2M_c^2e^{-z}[K_0(z) - K_1(z)]. \quad (13)$$

The correlator  $\Pi_{H-HH\pi\pi}^S(M_B)$  for  $2\pi$  decay from the hybrid component of the  $\Psi'(2S)$  to the  $J/\Psi(1S)$  state, obtained from the diagram in Fig.2, turns out to be less than 1 % of  $\Pi_{H\pi\pi}^S(M_B)$  of Fig. 1 and is not included.

Corresponding to the diagram shown in Fig.3, the correlator for the hybrid component of  $\Psi'(2S)$  to decay to a sigma and the  $J/\Psi(1S)$  state,  $\Pi_{H-HH\sigma}^{\mu\nu}(p)$ , is obtained using two results from previous work. First, it was shown in Ref[4] that the coupling of the mixed to the normal charmonium,  $\Pi_{H-HH}^S(M_B)$ , shown in Fig. 4(b), is given approximately by

$$\Pi_{H-HH}^S(M_B) \simeq \pi^2 \Pi_H^S(M_B) , \quad (14)$$

where  $\Pi_H^S(M_B)$  is the coupling of the normal component of  $\Psi'(2S)$  to the  $J/\Psi(1S)$  state.

Next we use the external field coupling shown in Fig.5(b) to relate  $\Pi_{H-HH\sigma}^{\mu\nu}(p)$  to the mixed coupling,  $\Pi_{H-HH}^S(M_B)$  which gives

$$\begin{aligned} \Pi_{H-HH\sigma}^S &= \frac{g_\sigma}{M_\sigma} < G^2 > \Pi_{H-HH}^S \\ &= \frac{g_\sigma}{M_\sigma} < G^2 > \pi^2 \Pi_H^S , \end{aligned} \quad (15)$$

from which we find

$$\Pi_{H-HH\sigma}^S(M_B) = \frac{24}{(4\pi)^2} g^2 \frac{g_\sigma}{M_\sigma} < G^2 > M_c^2 \pi^2 e^{-z} [K_0(z) - 4K_1(z)/3 + K_2(z)/3] , \quad (16)$$

where  $g_\sigma$  is the sigma-gluon coupling constant and  $< G^2 >$  is the gluon condensate.

From Eqs(13,16), with  $b=1/\sqrt{2}$  we obtain  $R$  = ratio of sigma to free two-pion (excluding the sigma) decay widths as

$$R = N \frac{\frac{g_\sigma}{M_\sigma} < G^2 > M_c^2 \pi^2 [K_0(z) - 4K_1(z)/3 + K_2(z)/3]}{g_\pi^2 [K_0(z) - K_1(z)]} , \quad (17)$$

with  $N$  a normalization factor derived below. The gluon-sigma coupling constant,  $g_\sigma$ , was found using a low-energy theorem[10] and has been used successfully in studying scalar glueballs[11], the production of sigmas in high-energy proton-proton collisions[12], and the decay of hybrid baryons[13]. The result is  $g_\sigma/M_\sigma \simeq 1.0$ . The pion quark coupling is known to be[14]  $g_\pi = f_\pi m_\pi^2/(m_u + m_d)$ . Using standard values for the quark masses,  $m_u = m_d = 3$  MeV,  $g_\pi^2 = 0.189 \text{ GeV}^4$ . The gluon condensate  $< G^2 > = 0.476 \text{ GeV}^4$ .

Finally, since the correlators do not use normalized states we must find the relative normalization of the normal and hybrid components. The normalization parameter is found from

$$N^2 = \frac{\int d(M_B^2) \Pi_H^S(M_B)}{\int d(M_B^2) \Pi_{H-HH}^S(M_B)} . \quad (18)$$

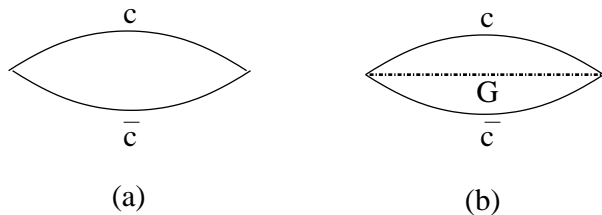


Figure 6: (a) Normal charmonium normalization (b) Hybrid charmonium normalization

Using Eq(10) for  $\Pi_H^S(M_B)$  and Eq(15) in Ref[5] for  $\Pi_{H-HH}^S(M_B)$  we find that  $N=0.0123 M_c^2$ . Using the charmonium mass,  $M_c$ , and the Borel mass,  $M_B$ =mass of the  $\Psi'(2S)$ , and  $(K_0(z), K_1(z), K_2(z))=(1.54, 3.75, 31.53)$ , we find for  $R$  = ratio of  $\sigma$  to  $2\pi$  decay of the  $\Psi'(2S)$  to the  $J/\Psi$

$$R = 0.98 . \quad (19)$$

This result is in very good agreement with recent measurements by the BES Collaboration at the IHEP, Beijing, [15]. The uncertainty in the  $R$  is not large, similar to the rapid convergence of diagrams used in QCD sum rules for heavy quark states. Since we are using only the lowest order diagram, from these previous calculations we estimate an uncertainty of about 15 percent, which was shown to arise mainly from the continuum contributions. This estimate of error is based on the method of QCD sum rules

### 3 Conclusion

Using the results of our previous study that found the  $\Psi'(2S)$  to be a mixed normal and hybrid charmonium state, we have found the branching ratio of the sigma to two-pion decays of this state to the  $J/\Psi$  to be in good agreement with BES measurements. With the successful application of this theory to explain the so-called  $\rho - \pi$  decay problem of the  $\Psi'(2S)$ [4], this gives further evidence that the  $\Psi'(2S)$  is a mixed heavy-quark hybrid. In future studies we shall explore other decays, and the production of the  $\Psi'(2S)$  in Relativistic Heavy Ion Collisions, where the octet model has been shown to be dominant[16, 17], consistent with the color 8 gluon of the hybrid component.

### 4 Acknowledgements

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## References

- [1] M.E.B Franklin et. al. (Mark II Collaboration), Phys. Rev. Lett. **51** 963 (1983)
- [2] F-K. Guo, P-N. Shen, and H-C. Chiang, Phys. Rev. **D 74**, 014011 (2006)
- [3] Z.Y. Zhou, G.Y. Qin, P. Zhang, Z.G. Xiao, H.Q. Zheng, N. Wu, JHEP 0502:043 (2005)
- [4] L.S. Kisslinger, Phys. Rev. **D 79**, 114026 (2009)
- [5] L. S. Kisslinger, D. Parno, S. Riordan, Adv.High Energy Phys. 2009:982341,2009
- [6] H. Vogel, hep-ex/060601, Proceedings of 4th Flavor Physics and CP Violation Conference (FPCP'06) (2006), [www.slac.stanford.edu/econf/C060409](http://www.slac.stanford.edu/econf/C060409)
- [7] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147**, 385; Nucl. Phys. **B147** 448 (1979)
- [8] C.B. Chiu, J. Pasupathy, and S.J. Wilson, Phys. Rev. **D32**, 1786 (1985)
- [9] E.M. Henley, W-Y.P. Hwang, and L.S. Kisslinger, Phys. Lett. **B367**, 21 (1996)
- [10] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B165** 67 (1980); Nucl. Phys. **B191** 301 (1981)
- [11] L.S Kisslinger and M.B. Johnson, Phys. Lett. **B523**, 127 (2001)
- [12] L.S. Kisslinger, W-h. Ma and P. Shen, Phys. Rev. **D 71**, 094021 (2005)
- [13] L.S. Kisslinger and Z. Li, Phys. Lett. **B445**, 271 (1999)
- [14] V.M. Dubovik and S.V.Zenkin, Ann. Phys. (NY) **172**, 100 (1986)
- [15] BES Collaboration, Phys. Lett. **B645** 19 (2007)
- [16] G.C. Nayak, M.X. Liu, and F. Cooper, Phys. Rev. **D 68**, 034003 (2003)
- [17] F. Cooper, M.X. Liu, and G.C. Nyak, Phys. Rev. Lett. **93**, 171801 (2004)